

Approaching Optimal Centralized Scheduling with CSMA-based Random Access over Fading Channels

Mehmet Karaca and Björn Landfeldt

Abstract—Carrier Sense Multiple Access (CSMA) based distributed algorithms can attain the largest capacity region as the centralized Max-Weight policy does. Despite their capability of achieving throughput-optimality, these algorithms can either incur large delay and have large complexity or only operate over non-fading channels. In this letter, by assuming arbitrary back-off time we first propose a fully distributed randomized algorithm whose performance can be pushed to the performance of the centralized Max-Weight policy not only in terms of throughput but also in terms of delay for completely-connected interference networks with fading channels. Then, inspired by the proposed algorithm we introduce an implementable distributed algorithm for practical networks with a reservation scheme. We show that the proposed practical algorithm can still achieve the performance of the centralized Max-Weight policy.

Index Terms—Max-Weight scheduling, CSMA, queue stability, distributed algorithm, fading channels.

I. INTRODUCTION

The major challenge of channel access management in wireless networks is optimally scheduling users so that interference is eliminated and the maximum performance in terms of throughput, delay, jitter, etc., can be achieved. The centralized Max-Weight algorithm (MW) [1] can achieve maximum throughput, and can stabilize the network. However, due to the huge complexity of MW algorithm Carrier Sense Multiple Access (CSMA) based algorithms have received significant attention from many researchers. In [2], [3], [4] and [5], CSMA based throughput optimal algorithms are developed for nonfading channels, which, however, suffer from either poor delay performance or high complexity. The authors in [6] present an optimal scheduling algorithm for deadline constrained-traffic by assuming continuous back-off time which is impossible to implement, and also the optimality does not hold for practical systems. Also, the performance of the discrete time version of the algorithm in [6] can be quite poor. The authors in [7] try to address the poor delay performance of CSMA based algorithms by assuming non-fading channels. The authors in [8] and [9] try to develop MW type algorithms for practical 802.11 networks without providing any analytical guarantees.

In order to be feasible and practical for wireless systems, any CSMA based algorithm should; i-) have low-complexity, and require only local information (no message passing); ii-

have good delay performance; iii-) perform well over general channel and arrival conditions. Since all these conditions can be satisfied by a centralized system with complete network information, we ask the following question: *Is it possible that a CSMA based distributed algorithm can achieve the same performance as the centralized solution in all these dimensions under certain conditions?* We find that the answer to this question is yes under the condition of a complete graph network. Our contributions are summarized as follows; i) when arbitrary back-off is allowed, we first develop an optimal distributed algorithm which schedules the user with the maximum weighted rate at every time slot without requiring any message passing over general fading and traffic conditions as the centralized MW algorithm does. Hence, the same performance as that of the centralized MW policy is indeed achievable; ii) Then, we design an implementable distributed algorithm to be compatible with practical systems, which can still schedule the user with the maximum weighted rate with low overhead associated with contention resolution.

II. SYSTEM MODEL

We consider a network where N users contend for transmission in the same contention domain. Time is slotted, $t \in \{0, 1, 2, \dots\}$, and each user's wireless channel is assumed to be independent across users and time. The gain of the channel is constant over the duration of a time slot but varies between slots. As in practice, we consider that only a fixed set of data rates $\mathcal{R} = \{r_1, r_2, \dots, r_L\}$ can be supported. The distribution of the channel rates for user n is denoted as $\Pr[R_n(t) = r_l] = p_n^l$, for all $l = \{1, \dots, L\}$, and $R_n(t) \in \mathcal{R}$ for all n and t . Let $I_n(t)$ be the scheduler decision, where $I_n(t) = 1$ if user n is scheduled for transmission in slot t , and $I_n(t) = 0$ otherwise. Each user maintains a separate queue, and packets arrive according a stationary arrival process that is independent across users and time slots. Let $A_n(t)$ be the amount of data arriving into the queue of user n at time slot t . Let $Q_n(t)$ and $R_n(t)$ denote the queue length and transmission rate of user n at time t , respectively. The queue length variation for user n is given as $Q_n(t+1) = [Q_n(t) + A_n(t) - R_n(t)I_n(t)]^+$, where $[y]^+ = \max(y, 0)$. Let $w_n(t)$ be the weighted rate of user n at time slot t , where $w_n(t) = Q_n(t)R_n(t)$. In their seminal paper, Tassiulas and Ephremides [1] have shown that Max-Weight algorithm scheduling the user $k = \operatorname{argmax}_n w_n(t)$ at every time slot can stabilize the network whenever this is possible, and we denote k as the user with the highest weighted rate at each slot.

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Unlike previous work, our aim is to develop a fully distributed algorithm that guarantees to schedule user k at every time slot. Next, we give our Distributed MW policy with Arbitrary Back-off Time (DMW-AB).

III. DMW WITH ARBITRARY BACKOFF

The basic principle behind our algorithm is to determine a back-off procedure in CSMA/CA (CSMA/Collision Avoidance) systems where users' back-off time is determined by their queue sizes and channel conditions. **DMW-AB** is performed in a two step procedure: In the first step, each user $n \in \{1, 2, \dots, N\}$ generates an exponentially distributed random variable denoted by $X_n(t)$ with rate $r_n(t) = f(w_n(t))$, which gives the random back-off duration of user n at that time slot. Then, if user n does not sense any other transmission until its random back-off duration expires, it starts transmitting. Otherwise it keeps silent.

Clearly, the performance of DMW-AB depends on the function $f(x)$. Next, we give our main theorem which will characterize $f(x)$ to make DMW-AB throughput-optimal. We note that our $f(x)$ function may not be unique, and there may be other functions that satisfy our theoretical results.

Theorem 1: Given any $b > 1$, if $f(x) = b^x$ and $R_n(t) \geq R_{min}$, and $R_{min} > 0$, $\forall n, t$ then DMW-AB is throughput-optimal.

Proof: The proof is provided in Appendix A. ■

Although for any $b > 1$, DMW-AB is optimal, the delay performance of it can be poor since Theorem 1 only holds when the queue sizes are sufficiently large. However, in Lemma 1 we will show that the condition on the queue sizes can become less restrictive for large values of b by guaranteeing to schedule the user k at every time slot.

Lemma 1: When $f(x) = b^x$ and as $b \rightarrow \infty$, the probability that the user with the maximum weighted rate is scheduled by DWM-AB algorithm at each time slot goes to 1.

Proof: The probability that the back-off timer of user k expires first under DWM-AB algorithm is given by $\pi_k(t) = \frac{b^{w_k(t)}}{\sum_{n=1}^N b^{w_n(t)}}$. By dividing both numerator and denominator by $b^{w_k(t)}$, we have,

$$\pi_k(t) = \frac{1}{1 + \sum_{\substack{n=1 \\ n \neq k}}^N b^{w_n(t) - w_k(t)}}$$

We know that $w_k(t) > w_n(t) \forall n \neq k$. Hence, taking $b \rightarrow \infty$ yields that $\lim_{b \rightarrow \infty} \pi_k = \frac{1}{1+0} = 1$. Thus, DMW-AB guarantees to schedule the user with the maximum weight. This completes the proof. ■

With Theorem 1 and Lemma 1, we show that DMW-AB can achieve the same performance as that of the centralized MW policy in terms of the average delay and throughput. Recall that DMW-AB algorithm assumes that users's back-off time can take any positive values. However, this assumption does not hold in practical systems. For instance, in current IEEE 802.11 networks a discrete-time backoff scale is used. Next, we introduce the practical version of DMW-AB.

IV. DMW WITH RESERVATION

In order to design a practical implementable algorithm, we divide each time slot into a control slot and a data slot whose

duration depends on the transmission duration at that time. The control slot consists of mini-slots that enable a collision-free data transmission. Let $M(t)$ be the number of mini-slots used for contention resolution at time t , which $M(t)$ is a random variable depending on the weighted throughput of users, and let $\mathbb{E}[M(t)]$ be the expected number of $M(t)$. Our practical algorithm namely DMW with Reservation Scheme (DMW-RS) is based on a reservation form, in which at the beginning of each mini-slot each user has to decide whether or not to attempt transmission in the current data slot. Let $a_n(m, t)$ be the attempt decision of user n at mini-slot m and time slot t . If user n attempts to transmit then $a_n(m, t) = 1$. Otherwise, $a_n(m, t) = 0$. Let $\tau(t)$ be a network parameter which determines the attempt decision of users as we will explain next.

The **DMW-RS** algorithm is performed as follows: First, at the beginning of each mini-slot m , each user $n \in \{1, 2, \dots, N\}$ generates an exponential random variable $X_n(m, t)$ with rate $r_n(t) = b^{w_n(t)}$. If $X_n(m, t) < \tau(t)$, then user n announces its intent by sending a short message (e.g., Request-to-Send Messages (RTS)), and $a_n(m, t) = 1$. Otherwise, user n stays idle (i.e., $a_n(m, t) = 0$) and skips to the next mini-slot. If more than one user attempts transmission at mini-slot m , there will be a collision. Also, if any user does not intend to transmit then that mini-slot will be idle. In case of a collision or idle mini slot, each user n generates a new exponential random variable with rate $r_n(t) = b^{w_n(t)}$, follows the same procedure and tries again. This process is iterated until there is only one user attempting to access the channel, and that user transmits its data in the data slot. We next show that DMW-RS can schedule the user with the maximum weighted throughput at every time. The probability that only the user k intends (e.g., only the user k sends RTS packet) and other users keep silent at a mini-slot is given by,

$$P_k(t) = \left(1 - e^{-\tau(t)r_k(t)}\right) \prod_{\substack{n=1 \\ n \neq k}}^N e^{-\tau(t)r_n(t)}. \quad (1)$$

One can show that $P_k(t)$ is a concave function of $\tau(t)$. Let $\tau^*(t)$ denote the value taken by $\tau(t)$ at the maximum value of $P_k(t)$. When all queue sizes, channel rates and b are given, $\tau^*(t)$ is found by taking the first derivative of $P_k(t)$, setting it to zero and solving for $\tau(t)$. Then, we have

$$\tau^*(t) = \frac{1}{r_k(t)} \ln \left(1 + \frac{r_k(t)}{\sum_{\substack{n=1 \\ n \neq k}}^N r_n(t)} \right). \quad (2)$$

When we plug $\tau^*(t)$ into equation (1), we obtain the maximum value of $P_k(t)$ denoted by $P_k^*(t)$ as follows:

$$P_k^*(t) = \left(\frac{\sum_{\substack{n=1 \\ n \neq k}}^N r_n(t)}{\sum_{n=1}^N r_n(t)} \right)^{\frac{\sum_{n \neq k}^N r_n(t)}{r_k(t)}} \left(\frac{r_k(t)}{\sum_{n=1}^N r_n(t)} \right). \quad (3)$$

The limit $\lim_{b \rightarrow \infty} P_k^*(t)$ can be directly evaluated by using L'Hospital's rule, and one can show that as $b \rightarrow \infty$ then $P_k^*(t) \rightarrow 1$. Hence, the user with the maximum weighted

throughput is guaranteed to be scheduled with DMW-RS.

Different from DMW-AB, DMW-RS introduces some amount of overhead in terms of mini-slots for contention resolution. Now, we turn our attention to develop an efficient method so that $\mathbb{E}[M(t)]$ is minimized as much as possible. We note that to minimize $\mathbb{E}[M(t)]$ we need to find $\tau^*(t)$. Clearly, in order to find the optimal $\tau^*(t)$, global knowledge of the network is required, i.e., $w_n(t)$ for all n . Next, our goal is to find a method for tracking and estimating $\tau^*(t)$ without requiring global network information.

In practice, users have finite buffer size. Let B^1 be the maximum buffer size of each user. Then, the maximum weighted rate that a user can achieve at any time slot is equal to $w_m = B \times r_L$ since at most B packets can be kept at each queue buffer and the maximum transmission rate is r_L . Let τ_l^* and τ_u^* be the minimum and maximum values that $\tau^*(t)$ can take at any time slot t , respectively. From (2), we approximate that τ_l^* is achieved nearly when $w_n(t) = w_m$ for all n . Thus, $\tau_l^* = \frac{1}{b^{w_m}} \ln \left(1 + \frac{1}{N-1} \right)$. Also, τ_u^* occurs when $w_n(t) = 0$ for all n . We note that this approximation is more accurate when N is sufficiently large. Then, $\tau_u^* = \ln \left(1 + \frac{1}{N-1} \right)$. The range of $\tau^*(t)$ is given as $\tau_l^* \leq \tau^*(t) \leq \tau_u^* \quad \forall t$.

Let $\tilde{\tau}(m, t)$ be the estimated² $\tau^*(t)$ at mini-slot m and time slot t , and let δ be the constant estimation parameter, and $\mathcal{B} = \{b_1, b_2, \dots, b_V\}$ be the set of b values, where $b_1 > 1$, and $b_g > b_h$, if $g > h$. The estimation follows two steps: first, we estimate $\tau^*(t)$ by decreasing $\tilde{\tau}(m, t)$ when a collision occurs at mini-slot m and increasing it if no user transmits. In the second step, if the number of consecutive collisions is higher than a threshold denoted by *collthr*, then decrease b to further reduce contention. If the number of consecutive idle slots is higher than a threshold denoted by *idlethr* then increase b so that the channel attempt probability increases. In Algorithm 1, the DMW-RS algorithm with updating τ is given. We note that the algorithm keeps tracking the number of consecutive collisions and idle slots, and checks if the boundary conditions for τ are satisfied. Also, in Step 3, at the beginning of next data slot, the algorithm uses the τ value at which the contention is resolved in the previous data slot. This is because if the network dynamics change slowly or never change (e.g., channel states and queue process change slowly from time slot t to time slot $t+1$), then one can expect that $\tau^*(t+1)$ will be similar to the value of $\tau^*(t)$ with high probability.

V. NUMERICAL RESULTS

In this section, we present our MATLAB simulation results. There are 20 users transmitting to an Access Point. We set $B = 200$ packets. Packets arrive independently at each slot according to a Poisson distribution for each user. The channel changes independently over users and time, we set $\mathcal{R} = \{1, 2, 3, 4, 5\}$ (i.e., $L = 5$) in packets. We consider

¹ B should be sufficiently large to avoid any throughput loss. In our algorithms since as b increases the condition on the maximum queue size in Theorem 1 becomes less restrictive, then the same performance can be achieved even with small buffer sizes.

²The estimation requires the knowledge of N , which can be done by each user online [10]. In this work, we assume that N is fixed, and known by each user.

Algorithm 1: DMW-RS with τ Update for user n at time t

- Step 0: At $t = 0$, every user sets $v = V$, $b = b_V$
 $\tilde{\tau}(1, t) = \frac{1}{b^\alpha} \ln \left(1 + \frac{1}{N-1} \right)$, $\alpha = w_m$,
 - Step 1: Then, at each mini slot m , user n generates an exponential random variable $X_n(m, t)$ with rate, $r_n(t) = b^{w_n(t)}$.
 - Step 2: If $X_n(m, t) < \tilde{\tau}(m, t)$, $a_n(m, t) = 1$. Otherwise $a_n(m, t) = 0$.
 - If $\sum_{n=1}^N a_n(m, t) > 1$, collision occurs. Then, update: $\alpha \leftarrow \alpha + \delta$, $\tilde{\tau}(m, t) \leftarrow \frac{1}{b^\alpha} \ln \left(1 + \frac{1}{N-1} \right)$, $m \leftarrow m + 1$. If $NumConsColl > collthr$, then $b \leftarrow b_{v-1}$. Go Step 1.
 - If $\sum_{n=1}^N a_n(m, t) = 0$, idle slot and update: $\alpha \leftarrow \alpha - \delta$. $\tilde{\tau}(m, t) \leftarrow \frac{1}{b^\alpha} \ln \left(1 + \frac{1}{N-1} \right)$, $m \leftarrow m + 1$. If $NumIdleColl > idlethr$, then $b \leftarrow b_{v+1}$ and Go Step 1.
 - Step 3: If $a_n(m, t) = 1$ and $a_k(m, t) = 0$ for all $k \neq n$, then user n transmits. Let τ_{prev}^* be the τ value at which the contention is resolved at time t . Set $t \leftarrow t + 1$, $\tilde{\tau}(1, t+1) \leftarrow \tau_{prev}^*$, and $b \leftarrow b_V$. Update the weight and Go Step 1.
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that users have heterogeneous channels, and we divide the users into two groups where there are ten users in each group. The channel state distributions of each group are given as follows: for the first group, $n = \{1, 2, \dots, 10\}$, $l = \{1, \dots, 5\}$, $p_n^l = \{0.15, 0.2, 0.2, 0.15, 0.3\}$, for the second group, $n = \{11, \dots, 20\}$, $l = \{1, \dots, 5\}$, $p_n^l = \{0.25, 0.25, 0.15, 0.1, 0.25\}$. We set $\delta=2$, *collthr* = *idlethr* = 7 for all m and t and for DMW-RS $\mathcal{B} = \{1.1, 1.2, 2\}$, and for DMW-AB $b = 2$. The simulation is run for 2×10^5 slots, which is sufficiently long for convergence of the queue sizes. We first evaluate the performance of the DMW-RS algorithm with Improved τ -update and DMW-AB in terms of the maximum supported traffic load and the average queue size, by comparing them with the centralized MW algorithm. For all algorithms, in Fig. 1(a) we plot the mean total queue size summed over all the users, as the overall arrival rate increases. Clearly, as the overall arrival rate exceeds approximately 5 packets/slot then queue sizes suddenly increase, and the network becomes unstable for all three algorithms, which also means that the achievable total throughput with these algorithms is equal to 5 packets/slot. In addition, the total average queue size is almost the same with all algorithms for every arrival rate. This result implies that DMW-AB and DMW-RS achieve almost the same performance in terms of the throughput and average delay as that of the centralized MW policy since DMW-AB and DMW-RS schedule the user with the highest weight at every slot with very high probability. Also, as b increases, the probability goes to one and, consequently, DMW-AB and DMW-RS achieve the same performance as that of the centralized Max-Weight policy.

In Fig. 1(b), for different values of δ we depict $\mathbb{E}[M(t)]$ with DMW-RS as the overall arrival rate increases when $N = 20$. It can be seen that for all values of δ and when the network

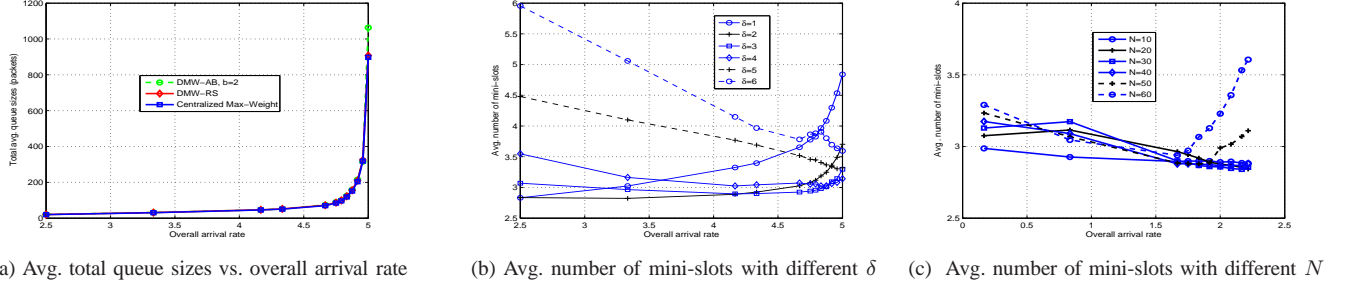


Fig. 1: Performance of DMW-RS

is highly loaded and even unstable, $\mathbb{E}[M(t)]$ tends to be less than 5 mini-slots, and DMW-RS is robust to the parameter δ . Lastly, in Figure 1(c), we depict $\mathbb{E}[M(t)]$ as the overall arrival rate increases with different number of users (i.e., N). When $N = 50$ and the arrival rate is equal to 1.8, and $N = 60$ with arrival rate of 1.6, the network is not stable, and at these points $\mathbb{E}[M(t)]$ starts increasing. However, $\mathbb{E}[M(t)]$ is less than 4 mini-slots for all simulation scenarios even if the channel and arrival processes are highly dynamic (i.e., independent over time.) and the number of users and the traffic rate are high.

VI. CONCLUSION

In this letter, we have proposed two new distributed scheduling policies for a fully-connected wireless network over fading channels. By allowing arbitrary backoff time we have proved that the first algorithm behaves exactly the same as the centralized Max-Weight policy. For practical systems, we then have developed a distributed algorithm that operates with a reservation scheme with a low number of mini-slots used for contention resolution, which can also achieve the same performance as the centralized Max-Weight policy. Our future directions include designing optimal algorithms to address the throughput-fairness and throughput-finite buffer trade-off and also investigating the adaptation of DMW-RS to practical systems such as 802.11 based networks.

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APPENDIX A PROOF OF THEOREM 1

We prove Theorem 1 using Theorem [11]. Let $w_{max}(t) = \max w_n(t)$ at time slot t . According to the Theorem in [11], if given any $0 < \epsilon, \delta < 1$, there exists a constant $B > 0$ such that: in any time with probability greater than $1 - \delta$, the complement of the following event χ occurs, where $\chi = \{l : w_l(t) < (1 - \epsilon)w_{max}(t)\}$. In order to show that $\pi(\chi') \geq 1 - \delta$, which implies Theorem 1, we next show that $\pi(\chi) = \sum_{l \in \chi} \pi_l < \delta$. Hence,

$$\pi(\chi) = \sum_{l \in \chi} \frac{b^{w_l(t)}}{\sum_{i=1}^N b^{w_i(t)}} \leq \frac{Nb^{(1-\epsilon)w^*(t)}}{\sum_{i=1}^N b^{w_i(t)}}$$

Note that $\sum_{i=1}^N b^{w_i(t)} \geq b^{w_{max}(t)}$. Therefore, we have

$$\pi(\chi) \leq \frac{Nb^{(1-\epsilon)w_{max}(t)}}{\sum_{i=1}^N b^{w_i(t)}} \leq \frac{Nb^{(1-\epsilon)w_{max}(t)}}{b^{w_{max}(t)}} \quad (4)$$

By using (4), one can show that if the following inequality in (5) holds then $\pi(\chi) < \delta$:

$$w_{max}(t) > \frac{\log_b(N) + \log_b(\frac{1}{\delta})}{(\epsilon)} \quad (5)$$

Since $w_{max}(t)$ is a continuous and nondecreasing function of queue sizes, and $R_n(t) \geq R_{min} > 0$, with $\lim_{Q(t) \rightarrow \infty} w_{max}(t) = \infty$, there exists $B > 0$ such that

$$w_{max}(t) > \max_n Q_n(t) R_{min} > \frac{\log_b(N) + \log_b(\frac{1}{\delta})}{(\epsilon)} \quad (6)$$

Hence, if queue sizes are large enough we can find a constant B such that

$$\max_n Q_n(t) > \frac{\log_b(N) + \log_b(\frac{1}{\delta})}{(R_{min})\epsilon} \triangleq B \quad (7)$$

Hence, Theorem 1 holds and $\pi(\chi) < \delta$. Thus, DMW-RS is throughput-optimal.